A stochastic collocation approach to constrained optimization for random data identification problems

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ABSTRACT

In this talk we will present a scalable, embarrassingly parallel mechanism for optimal identification of statistical moments (mean value, variance, covariance, etc.) or even the whole probability distribution of input random data, given the probability distribution of some response (quantities of physical interest) of a system of stochastic partial differential equations (SPDEs). This stochastic inverse problem can be described by an objective functional constrained by a system of parameterized PDEs. Given several objectives we prove the existence of an optimal solutions, establish the validity of the Lagrange multiplier rules and obtain stochastic optimality systems of equations. To characterize data with moderately large amounts of uncertainty we introduce a novel stochastic parameter identification algorithm that integrates an adjoint-based deterministic algorithm with our adaptive sparse grid stochastic collocation FEM. This allows for decoupled, moderately high dimensional, parameterized computations of the stochastic optimality system, where at each collocation point, deterministic analysis and techniques can be utilized. Our rigorously derived error estimates, for the fully discrete problems, will be described and used to compare the efficiency of the method with several other techniques. Numerical examples illustrate the theoretical results and demonstrate the distinctions between the various stochastic identification objectives.