Discontinuous Galerkin Finite Element Methods for High Order Nonlinear Partial Differential Equations

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Discontinuous Galerkin (DG) finite element methods were first designed to solve hyperbolic conservation laws utilizing successful high resolution finite difference and finite volume schemes such as approximate Riemann solvers and nonlinear limiters. More recently the DG methods have been generalized to solve convection dominated convection-diffusion equations (e.g. high Reynolds number Navier-Stokes equations), convection-dispersion (e.g. KdV equations) and other high order nonlinear wave equations or diffusion equations. In this talk we will first give an introduction to the DG method, emphasizing several key ingredients which made the method popular, and then we will move on to introduce a class of DG methods for solving high order PDEs, termed local DG (LDG) methods. We will highlight the important ingredient of the design of LDG schemes, namely the adequate choice of numerical fluxes, and emphasize the stability of the fully nonlinear DG approximations. Numerical examples will be shown to demonstrate the performance of the DG methods.